№ 1

$$\begin{cases} u_{tt} = a^2 \Delta_2 u & 0 \le r < r_0, \quad 0 \le \varphi < 2\pi \quad t > 0 \\ u|_{t=0} = 0, & u_t|_{t=0} = J_3\left(\frac{\mu_5}{r_0} r\right) \cdot \cos 3\varphi \\ u|_{r=r_0} = 0 \end{cases}$$

$$J_3(\mu_5) = 0$$

$$u = W(r, \varphi) \cdot T(t)$$

$$\frac{T''}{a^2 T} = \frac{\Delta_2 W}{W} = -\lambda$$

1) Будем искать W в виде $W = R(r) \cdot \cos 3\varphi$

$$\Delta_2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) + \frac{1}{r^2}\frac{d^2}{d\varphi^2}$$

т.о. $\frac{1}{r}(R' + rR'') \cdot \cos 3\varphi - \frac{9}{r^2} R \cos 3\varphi + \lambda R \cos 3\varphi = 0$

Делим на $\cos 3\varphi$, $*$ на r^2, группируем:

$$r^2 R'' + rR' + (\lambda r^2 - 3^2) R = 0$$

$n = 3$, замена $\sqrt{\lambda} r = x$

. . .

$$R_k = J_3\left(\frac{\mu_k}{r_0} r\right), \text{ где } \mu_k : J_3(\mu_k) = 0$$

$$W_k = J_3\left(\frac{\mu_k}{r_0} r\right) \cdot \cos 3\varphi$$

$$\lambda_k = \left(\frac{\mu_k}{r_0}\right)^2$$

2) $T(t) = A_k \cos\left(a \frac{\mu_k}{r_0} t\right) + B_k \sin\left(a \frac{\mu_k}{r_0} t\right)$

3) $u(r, \varphi, t) = \sum_{k=1}^{\infty} J_3\left(\frac{\mu_k}{r_0} r\right) \cos 3\varphi \left(A_k \cos\left(a \frac{\mu_k}{r_0} t\right) + B_k \sin\left(a \frac{\mu_k}{r_0} t\right)\right)$

$$u|_{t=0} = 0 \Rightarrow A_k = 0.$$

$$u_t|_{t=0} = J_3\left(\frac{\mu_5}{r_0} r\right) \cos 3\varphi$$

$$u_t|_{t=0} = \sum_{k=1}^{\infty} J_3\left(\frac{\mu_k}{r_0} r\right) \cos 3\varphi \, B_k \, a \frac{\mu_k}{r_0}$$

$\Rightarrow B_5 = \frac{r_0}{a\mu_5}$, остальные B_k равны 0.

Ответ: $u(r, \varphi, t) = J_3\left(\frac{\mu_5}{r_0} r\right) \cos 3\varphi \cdot \frac{r_0}{a\mu_5} \cdot \sin\left(a \frac{\mu_5}{r_0} t\right)$

D/з 8 2/2

№2

$$\begin{cases} u_{tt} = \Delta_2 u & 0 \le r < 3 \quad 0 \le \varphi < 2\pi \quad t > 0 \\ u|_{t=0} = 5 J_0\left(\frac{\mu_5}{3} r\right) & u_t|_{t=0} = 0 \\ u|_{r=3} = 0 & \end{cases}$$

$J_0(\mu_5) = 0$

1) $u = W(r, \varphi) \cdot T(t)$

$$\frac{\Delta_2 W}{W} = \frac{T''}{T} = -\lambda$$

2) Будем искать W в виде $W = R(r)$

$\Delta_2 W = \frac{1}{r}(R' + rR'')$

$\frac{1}{r}(R' + rR'') + \lambda R = 0$

$r^2 R'' + rR' + \lambda r^2 R = 0$

$n = 0$. ур-ие Бесселя

$R = J_0\left(\frac{\mu_k}{3} r\right)$, где $J_0(\mu_k) = 0$

$\lambda_k = \left(\frac{\mu_k}{3}\right)^2$

3) $T(t) = A_k \cos\left(\frac{\mu_k}{3} t\right) + B_k \sin\left(\frac{\mu_k}{3} t\right)$

4) $u = \sum\limits_{k=1}^{\infty} J_0\left(\frac{\mu_k}{3} r\right) \cdot \left(A_k \cos\left(\frac{\mu_k}{3} t\right) + B_k \sin\left(\frac{\mu_k}{3} t\right)\right)$

$u_t|_{t=0} = \sum\limits_{k=1}^{\infty} J_0\left(\frac{\mu_k}{3} r\right) B_k \frac{\mu_k}{3} = 0$

$\Rightarrow B_k = 0$

$u|_{t=0} = \sum\limits_{k=1}^{\infty} J_0\left(\frac{\mu_k}{3} r\right) \cdot A_k = 5 J_0\left(\frac{\mu_5}{3} r\right)$

$\Rightarrow A_5 = 5 \quad A_k = 0$. остальные

Ответ: $u(r, \varphi, t) = 5 J_0\left(\frac{\mu_5}{3} r\right) \cdot \cos\left(\frac{\mu_5}{3} t\right)$